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Experimental observation of resonances in modulated turbulence

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The response of turbulence to a periodic forcing of the Reynolds number is studied. The turbulent flow is produced in a closed geometry between two counter-rotating disks. The time series of both the global injected power and the local velocity are measured. It is found that the injected power fluctuations exhibit resonances for well-determined values of the modulation frequency, making it possible to estimate a turbulent cascade time. For modulation periods larger than the measured cascade time, the response of the velocity fluctuations is simply proportional to the modulation of the forcing. For high-frequency modulations, the velocity fluctuations are strongly damped: their amplitude decreases as the inverse of the modulation frequency.

1. Introduction

Turbulence is a problem of immense practical importance, but some of the most basic questions about it still remain to be answered (Sreenivasan & Antonia 1997). In the usual cascade picture of turbulence, kinetic energy is injected at large scales and transported through the energy cascade towards smaller scales, at which it dissipates (Richardson 1926; Kolmogorov 1941). The dynamics of this transfer remain a challenge in the fundamental comprehension of turbulence. For instance, there is no precise information about the time scales involved in this transfer. Only for the particular case of decaying turbulence have time scales been measured and scaling laws with the Reynolds number proposed (Smith *et al.* 1993). For stationary forced turbulence, the typical cascade time scale can be easily estimated by direct numerical simulation from the time delay between the global instantaneous kinetic energy and enstrophy (see Pumir 1996). In experiments, however, such instantaneous measurements are not yet possible.

In order to be able to estimate the cascade time experimentally, Lohse (2000) proposed recently of study ‘kicked’ turbulence, in which the response of the turbulent flow to a periodic external forcing is examined. Heydt, Grossmann & Lohse (2003) extended this study, and investigated the case of a small modulation of energy injection theoretically, using a mean-field type of approach. They found that for slow modulations (having a period larger than the characteristic time of the energy cascade), the system simply follows the driving modulation with the same amplitude.

The periodic forcing merely makes the Reynolds number of the flow vary periodically, too. The more interesting finding was that, for modulation periodicities that are smaller than the typical cascade time, the response amplitude of the system decreases and resonances are observed in the response of the system. The resonances, if observable experimentally, therefore contain information about the typical time scale of energy transfer in the cascade.

Besides the fundamental interest of this situation for the understanding of turbulence, many natural or engineering turbulent flows are driven periodically. Classical examples are geophysical flows periodically driven by tides and seasons, blood flows periodically driven by the heart, and flows in combustion chambers of engines. The effects of flow modulation of Poiseuille-type flows in either smooth pipes or channels have been studied extensively (e.g. Gerrard 1971; Ramaprian & Tu 1983; Mao & Hanratty 1986; Lodhal, Sumer & Fredsoe 1998 and references therein). However, these studies address the effect of modulation on a turbulent boundary layer. This is very different from bulk turbulence, to which the theories of Lohse (2000) and Heydt *et al.* (2003) apply. More generally, the effect of a large-scale modulation on turbulence remains an almost unresolved issue since most theoretical studies address the problem of constant energy injection (Sreenivasan & Antonia 1997).

In this paper, we study the effect of a modulation of the integral-scale Reynolds number on turbulence using a set-up for which the global energetic properties of the flow are given by the bulk turbulence only (Cadot *et al.* 1997): the dissipation in the boundary layers near the walls is negligible. This is achieved using a closed cylindrical cell filled with water; the turbulence is generated using two counter-rotating stirrers (Labbé, Pinton & Fauve 1996; Titon & Cadot 2003), one at each end of the cylinder. It is the rotation frequency of the stirrer that gives the Reynolds number of the flow, and by modulating the rotation frequency, the Reynolds number is modulated.

We provide experimental evidence for the existence of resonances in modulated turbulence that have, to the best of our knowledge, not been observed before. The results compare satisfactorily to the theoretical investigations of Heydt *et al.* (2003) which allow the computation of the turbulent cascade time. This article is organized in three sections. the next section briefly presents the experimental set-up and the measurement technique. The results for both the dynamical and the static response constitutes the § 3. Finally, in the last section we provide a discussion and conclusions about the time scales found in the turbulence response.

2. Experimental set-up

In experiments on turbulence, flows are generally driven either at constant velocity or by a constant force. For instance, pipe flows are realized by either setting the flow rate (constant velocity forcing) or the pressure gradient (constant force forcing). In both cases, the energy injection is not constant, and the injected power randomly fluctuates due the feedback of the turbulence produced on the forcing (Titon & Cadot 2003). We first measure the injected power. To characterize the turbulent flow, we then measure the velocity (and its fluctuations) in the turbulent bulk of the flow.

Titon & Cadot (2003) have described the experimental cell in detail. The turbulence is generated in a closed cylindrical cell ($V = 11\,\mathrm{l}$) between two counter-rotating stirrers (disks with blades) of radius $R = 8.75\,\mathrm{cm}$ spaced two disk diameters apart. A DC servomotor regulated by a servo amplifier (Parvex) drives each stirrer independently. The motors are configured to keep the disks rotating at a fixed frequency, independently of the torque exerted by the turbulence on the disks. This is done by using a tachymetry feedback loop, a regulation system that changes the torque delivered by the motors to maintain the imposed angular velocity. The time response of the control loop is 0.05 s, implying a high-frequency cutoff of 20 Hz (see Titon & Cadot 2003 for further details). For lower frequencies, the image of the current at the output of the feedback loop gives an instantaneous measurement of the torque. The mechanical power supplied by the motors is simply given by the product of the current at the feedback output (i.e. torque) and the tachymetry signal (i.e. angular velocity).

The total power injected into the flow $P(t)$ is computed by summing the contribution of each servomotor's mechanical power:

$$P_1^{mech}(t) = \Omega_1(t)\Gamma_1(t), \quad P_2^{mech}(t) = \Omega_2(t)\Gamma_2(t), \tag{2.1}$$

and subtracting the power due to the torques of friction losses $\Gamma_1^s(\Omega_1)$ and $\Gamma_2^s(\Omega_2)$ which were measured independently in the empty cell (full of air), as well as the inertia of the forcing devices, for which a correction is necessary. We have

$$P(t) = P_1^{mech}(t) + P_2^{mech}(t) - \Omega_1(t)\Gamma_1^s - \Omega_2(t)\Gamma_2^s - \frac{1}{2}I_1\frac{\Omega_1^2(t)}{\mathrm{d}t} - \frac{1}{2}I_1\frac{\Omega_2^2(t)}{\mathrm{d}t}, \tag{2.2}$$

where Ω_1 and Ω_2 denote the angular velocities of the two forcing devices. In the following, we use the injected power per unit mass defined as $\epsilon_I(t) = P(t)/M$ where M is the total mass of water in the cell.

We utilize a hot-film probe (Dantec) to measure the modulus u of the local velocity; its square is a measure of the local turbulence kinetic energy. The probe is placed halfway between the stirrers, 3 cm from the wall in the bulk of the flow. We cool the water in the cell between data acquisitions so that the temperature of the fluid is in the range $20 < T < 21^\circ$C, in order to have reliable hot-film measurements.

The flow is continuously stirred, the two stirrers being in counter-rotation. The Reynolds number for the stationary regime is defined as $Re = \Omega_0 R^2/\nu$, where $\Omega_0/2\pi$ is the rotation frequency of the stirrers and ν the kinematic viscosity of the fluid. For the present experiment, the flow is fully turbulent: $\Omega_0/2\pi = 5$ Hz and $Re = 240\,000$. The corresponding microscale Reynolds number is $R_\lambda \sim 650$ (see Arneodo *et al.* 1996 for a study of the small-scale velocity statistics in the present flow configuration). A modulation of amplitude $a\Omega_0$ and frequency f_m of the forcing is applied:

$$\Omega_1(t) = -\Omega_2(t) = \Omega(t) = \Omega_0(1 + a\cos(2\pi f_m t)). \tag{2.3}$$

We vary the modulation frequency from 0 to 20 Hz with a constant amplitude of $a = 0.1$. In addition, we vary the modulation amplitude from 0.05 to 0.34 for two frequencies of modulation: 0.5 Hz and 5 Hz.

The rotation frequency $\Omega(t)$, the torques, $\Gamma_1(t)$ and $\Gamma_2(t)$ of each motor, and the hot-film probe signal are simultaneously recorded by a data acquisition board with a sampling rate of 5 kHz. For each modulation frequency f_m we recorded a file of 1.5 million points corresponding to a duration of 300 s.

3. Results

3.1. *Dynamical response*

To study the dynamical response of the system, we correlate the fluctuations of the energy injection rate $\epsilon_i(t)$ and the local velocity modulus $u(t)$ with those of the modulated rotation frequency $\Omega(t)$. In the following, we consider the normalized

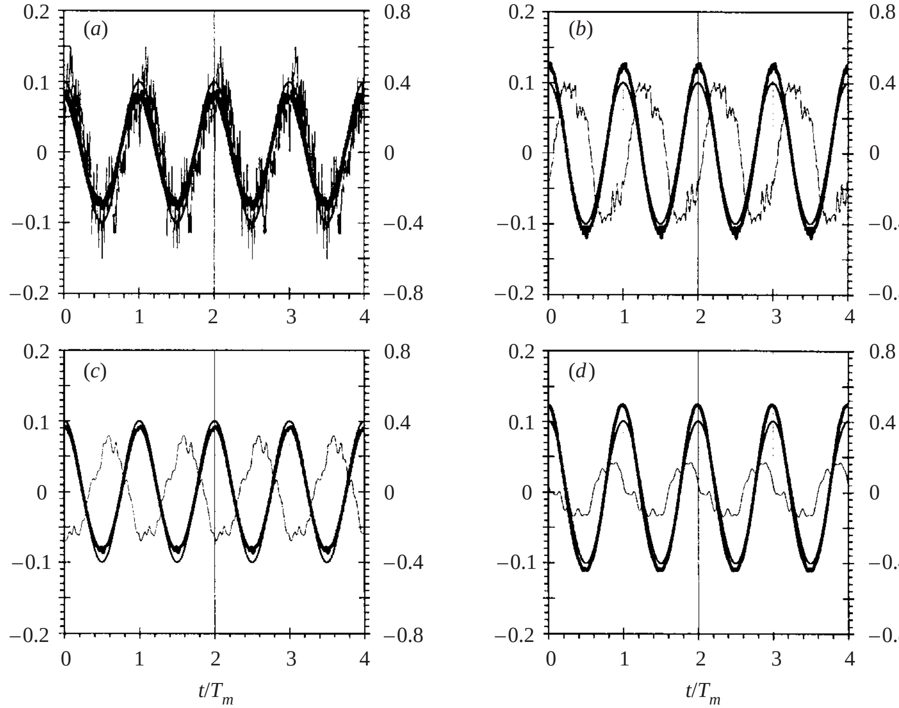

FIGURE 1. Phase average of: the normalized velocity modulus fluctuations $\delta u'$ (thin line, left vertical axis); the normalized angular velocity fluctuations $\delta\Omega'$(medium thick line, left vertical axis); the normalized injected power fluctuations $\delta\epsilon_i'$ (thick line, right vertical axis). (*a*) $f_m = 0.5\,\mathrm{Hz}$; (*b*) $f_m = 2\,\mathrm{Hz}$; (*c*) $f_m = 3.5\,\mathrm{Hz}$; (*d*) $f_m = 5\,\mathrm{Hz}$. Phase averages are taken over one period $T_m = 1/f_m$ of the angular velocity of the stirrers and are repeated four times for better accuracy of the results.

fluctuations defined as

$$\delta x'(t) = \frac{x(t) - \langle x \rangle}{\langle x \rangle}, \tag{3.1}$$

where x is either ϵ_i, u, or Ω, and $\langle\,\rangle$ denotes the time-averaged quantity.

Figure 1 shows a direct comparison between $\delta\Omega'$, $\delta\epsilon_i'$ and $\delta u'$ (obtained by averaging over one modulation period) for different modulation frequencies: 0.5, 2, 3.5 and 5 Hz. It is seen that for low frequencies of modulation, the signals are modulated in the same way; both $\delta u'$ and $\delta\epsilon_i'$ simply follow the slow modulation of the rotation frequency. For higher modulation frequencies, the response of the velocity fluctuations reduces considerably: the turbulent system is no longer able to follow the modulation. The phase difference between the injected power and the velocity shifts by a value close to $\pi/2$ between each of the modulation frequencies shown in figure 1.

In order to quantify the effect of the modulation on the turbulence, we perform a Fourier analysis and extract the modulation mode from each time series. We treat the data in the following way. The time series $x(t)$ is separated in five blocks of 300 000 points each. A fast Fourier transform is computed on each block (the frequency resolution is 0.017 Hz). Characteristic amplitude spectra of the velocity $A_u(f)$ and the injected power $A_{\epsilon_i}(f)$ are shown in figure 2. For both cases, turbulence modulated at $f_m = 2\,\mathrm{Hz}$ is compared to the unmodulated turbulence. For both the injected power and the velocity, modulation of the turbulence leads to the appearance

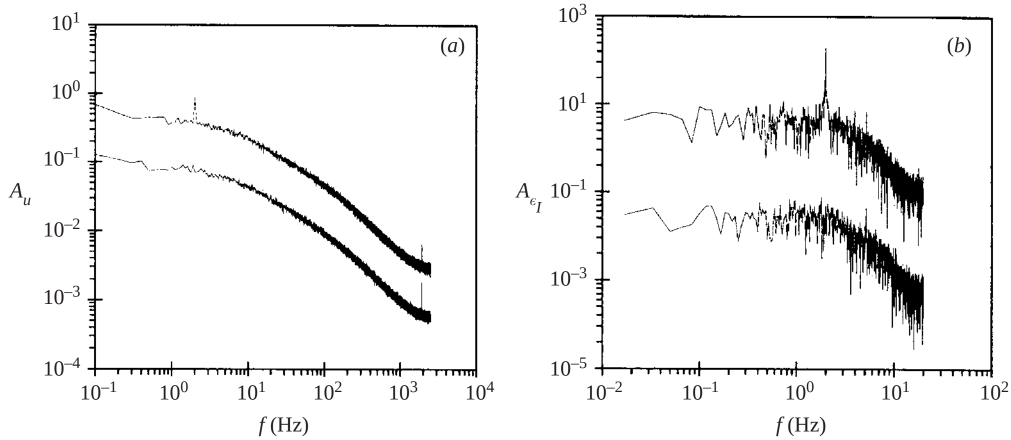

FIGURE 2. Amplitude spectra of (*a*) the velocity modulus and (*b*) the injected power for modulated (at $f_m = 2$ Hz) and unmodulated turbulence. For comparison, the modulated case is shifted up by one decade. We only show the data for frequencies lower than 20 Hz in (*b*) for which the power measurements are reliable. The form of the spectra is different for the velocity (*a*) and the power (*b*) because the injected power is a global quantity; consequently its spectrum decreases quickly beyond the mean rotation frequency: $\Omega_0/2\pi = 5$ Hz (Labbé *et al.* 1996; Titon & Cadot 2003). The amplitude spectrum for the velocity extends up to the dissipative scale of the turbulence, but the dissipative scale is not resolved due to the cut-off frequency corresponding to the probe size. Note as well that the velocity measurements are performed at a location for which the Taylor hypothesis is not valid; therefore we do not retrieve the scaling laws usually found for the spectrum (Sreenivasan & Antonia 1997).

of a clear peak around the modulation frequency in the spectrum. The amplitude of the modulation mode $A_x(f_m)$ is measured directly from the peak of the amplitude spectrum $A_x(f)$. The phase at the modulation frequency, $\Phi_x(f_m)$ is obtained from the phase spectrum of $x(t)$; the phase of the rotation frequency of the stirrers is taken as the origin. Finally, the mean value and the statistical error are computed from the values of amplitudes and phases obtained from the 5 different blocks.

Dimensionless response amplitudes $\bar{A}_x(f_m)$ are normalized by the time-averaged signal defined as

$$\bar{A}_x(f_m) = \frac{A_x(f_m)}{\langle x \rangle}. \tag{3.2}$$

The amplitude response for the power and the local velocity is displayed in figure 3. The amplitude of the angular velocity of course remains constant around the imposed value: $\bar{A}_\Omega = a = 0.1$, shown as the solid line. The amplitude of the local velocity $\bar{A}_u$ also has a constant magnitude close to 0.1, but only for modulation frequencies lower than $f_{cross} \approx 3$ Hz, implying that for these frequencies the turbulent velocity field follows the imposed modulation. However, for higher frequencies, the response amplitude is damped as $1/f_m$ (figure 3*a*, inset).

A surprising result is that the amplitude response of the power in figure 3(*b*) is not monotonic and presents two clear maxima at 2 Hz and 5 Hz separated by a minimum at 3.5 Hz.

The phase shift ϕ_u between the local velocity and the imposed modulation is shown in figure 4. It decreases linearly with increasing modulation frequency. This linear behavior defines an intrinsic time scale τ_{int}: $\phi_u(f_m) = 2\pi\tau_{int} f_m$, so that we find $\tau_{int} \approx 0.25$ s. The phase shift between the injected power and the imposed modulation

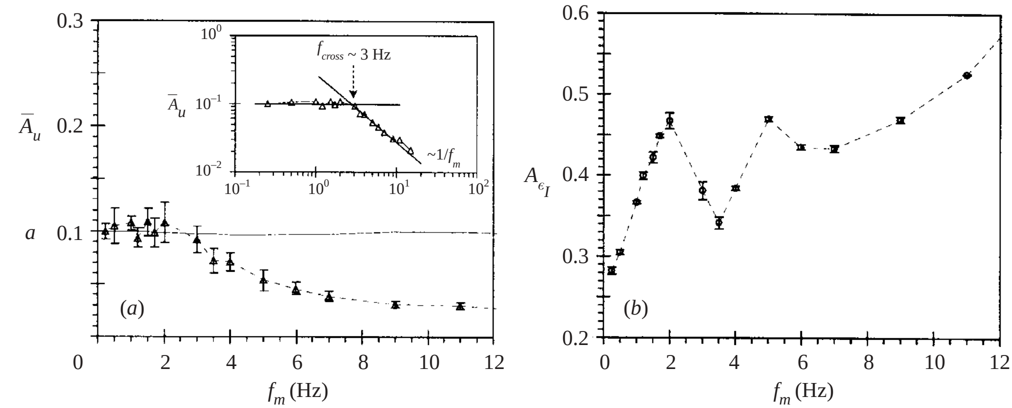

FIGURE 3. Dimensionless amplitude response at the modulation frequency vs. the modulation frequency of (a) rotation frequency of the disks (continuous line) and local velocity modulus (triangles, inset: log-log plot of the velocity modulus.); (b) injected power (open circles).

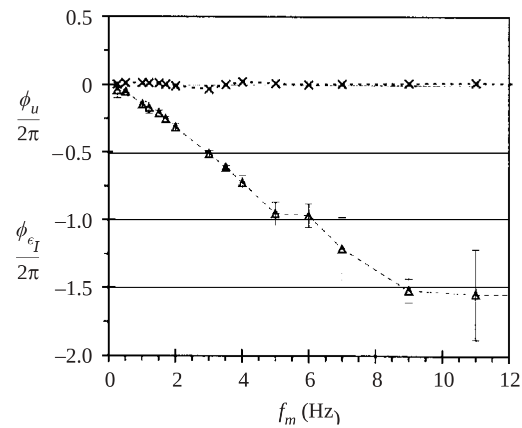

FIGURE 4. Phase response at the modulation frequency vs. the modulation frequency of the local velocity modulus (triangles) and the injected power (crosses).

presents a very small oscillation. The phase shift is close to zero in the vicinity of the maxima of the power amplitude at (at 2 and 5 Hz) and takes a negative value for the minimum of the power amplitude (at 3.5 Hz). However, in comparison to the phase shift between the local velocity and the forcing, this effect is small.

So far we have quantified the amplitude and phase of the response for a given modulation amplitude. We now investigate the response to a change in the magnitude of the modulation amplitude. It is found that the response usually varies linearly with the amplitude of modulation. This is shown in figure 5 for modulation frequencies 0.5 and 5 Hz. The amplitude response to the velocity modulus varies linearly up to large values of the modulation amplitude (figure 5b). Note that the dependence cannot be linear around zero because the amplitude response of turbulence at f_m is not zero without modulation (see the spectra for unmodulated turbulence in figure 2).

The phase shift of the velocity remains constant for all values of the modulation amplitude. These results also show that the intrinsic time scale found in figure 4 is not affected by changing the modulation amplitude. On the other hand, the amplitude

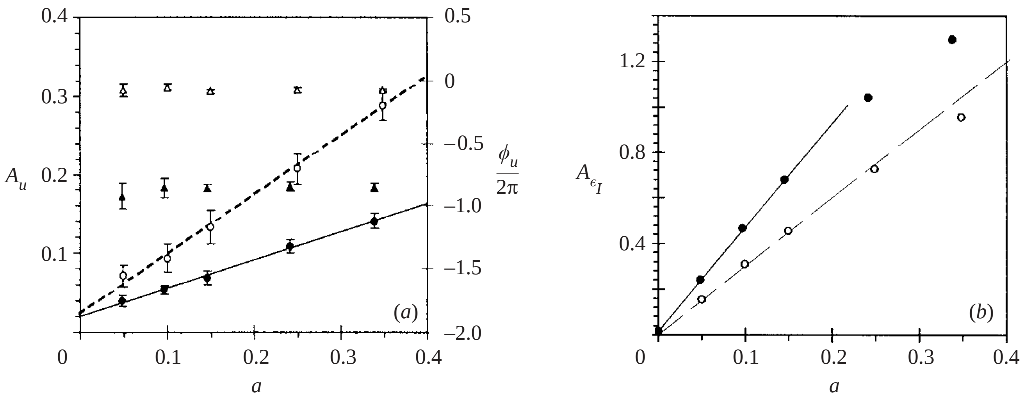

FIGURE 5. Amplitude and phase response vs. the modulation amplitude for modulation frequencies of $f_m = 0.5$ Hz (open symbols) and $f_m = 5$ Hz (filled black symbols). (*a*) Amplitude response (circles) with linear approximation, and phase response (triangles) of the velocity modulus. (*b*) Amplitude response of the injected power with linear approximations. The linear approximation of the modulation at $f_m = 0.5$ Hz (dashed line) has a slope of 3 (see discussion).

response of the injected power (figure 5*b*) remains linear only as long as $a < 0.2$. For larger values of the modulation amplitude a nonlinear saturation is observed.

3.2. *Global response*

We now focus on the effect of the modulation with a fixed amplitude $a = 0.1$ on the global (time-averaged) response of the system. We investigate the behaviour, as a function of the modulation frequency, of both the average injected power and the local turbulent kinetic energy, the latter being given by the square of the measured velocity.

Figure 6 shows the mean global injected power $\langle \epsilon_I \rangle$ and the mean turbulent intensity $\langle u^2 \rangle$ as a function of the modulation frequency. Globally, the modulation results in an increase in the mean injected power. However, around 3.5 Hz, we observe a region for which the modulation does not result in a significant increase. This minimum corresponds to the minimum in the amplitude response observed in figure 3(*b*).

For the kinetic energy, although the scatter of the measurements is rather large, we can still observe that the region around 3.5 Hz corresponds to a global increase of the turbulent energy. In addition, the data of figure 6(*a*) are strongly correlated with the data of figure 6(*b*): each increase (resp. decrease) of injected power coincides with a decrease (resp. increase) of the kinetic energy. Since the uncertainty in the kinetic energy is small compared to the scatter of the data points, it is possible that there is an additional fine structure in figure 6(*b*), whose origin could be related to the eight blades fitted on each stirrer.

4. Discussion and conclusion

We have measured the phase and amplitude response of a turbulent system subjected to a modulation of the Reynolds number. Here, we first compare our measurements to dimensional arguments deduced for non-modulated turbulence. We subsequently interpret the different measured turbulence time scales in the light of the theory of Heydt *et al.* (2003). Finally we discuss the consequences of the modulation for the mean dissipation and mean kinetic energy of the turbulence.

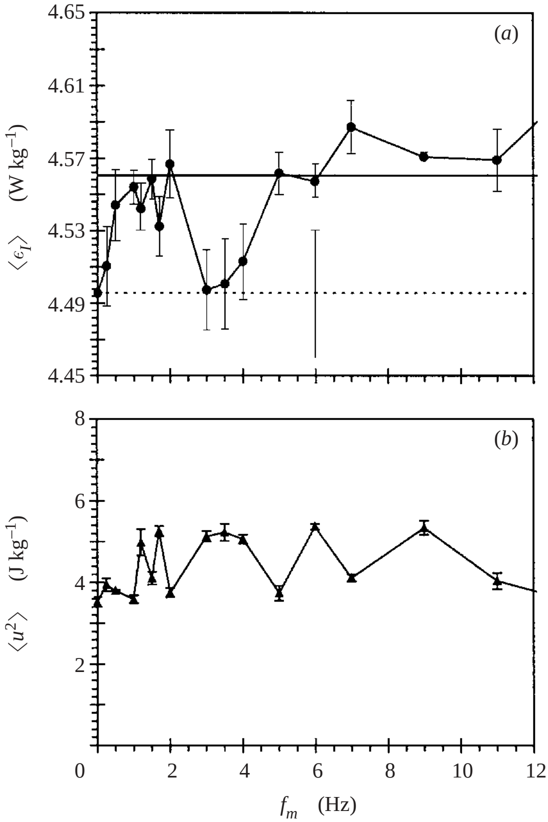

FIGURE 6. (*a*) Mean global injected power per unit mass vs. the modulation frequency. The solid line represents the corrected power expected by a simple dimensional argument of equation (4.2). The dashed line represents the measured injected power $\langle \epsilon_0 \rangle$ without any modulation. (*b*) Mean local turbulent intensity vs. the modulation frequency.

For the stationary non-modulated regime, the injected power is proportional to the cube of the angular velocity (Cadot *et al.* 1997) as expected from the (Kolmogorov 1941) dimensional arguments. Naively, one therefore expects that for the modulated system:

$$\epsilon_I(t) = \epsilon_0(1 + a\cos(2\pi f_m t))^3, \tag{4.1}$$

where ϵ_0 denotes the mean injected power without modulation. From this expression, the mean injected (or dissipated) power should be

$$\langle \epsilon_I \rangle = \epsilon_0(1 + 3a^2/2). \tag{4.2}$$

These estimates give a good order of magnitude for the turbulence response. On figure 6(*a*), equation (4.2) describes the average value of the power satisfactorily. The dynamical response of the injected power at the low modulation frequency $f_m = 0.5$ Hz (figure 5*b*) also corresponds to the value expected from the simple argument of equation (4.1). In this expression, the amplitude for the modulation mode f_m, is $\bar{A}_{\epsilon_I}(f_m) = 3a + o(a^2)$. However, the simple dimensional arguments given above for the injected power cannot explain the non-trivial response resulting in the existence of extrema in both injected power (figure 3*b*, figure 6*a*) and kinetic energy response (figure 6*b*). We now discuss the different timescales found in the measurements.

In the experiment, the forcing is applied at the stirrers only and not homogeneously in the bulk. A consequence is that, in addition to the cascade time, there is a large-scale convective time due to the momentum transport from the stirrers to the bulk of the flow. This effect then introduces a time lag between the modulation at the stirrers and the local modulation at the location of the hot-film probe. This time lag depends on the mean flow, i.e. on Ω_0. The phase shift ϕ_u between the local velocity and the imposed modulation shown in figure 4 decreases linearly with increasing modulation frequency. This behaviour defines a constant time lag which does not depend on the modulation frequency. Thus, this time lag, which we defined above as an intrinsic time scale for the turbulence $\tau_{int} \approx 0.25\,\mathrm{s} = \frac{5}{4} 2\pi/\Omega_0$, is consistent with a large-scale convection time.

A second time scale for the turbulence can be deduced from the local velocity response (figure 3*a*). We find that the response amplitude is constant for modulation frequencies lower than $f_{cross} = 3\,\mathrm{Hz}$. For higher frequencies, a low-pass filtering occurs and the response amplitude decreases as $(1/f_m)$: the velocity no longer follows the imposed modulation. The crossover frequency between the constant response and the $(1/f_m)$ decrease defines a characteristic time for the stationary turbulence: $\tau_0 = 1/f_{cross} \approx 0.33\,\mathrm{s} = \frac{5}{3} 2\pi/\Omega_0$.

The low-pass filtering behaviour observed in the velocity fluctuations is in very good agreement with the theoretical expectations of Heydt *et al.* (2002). They predict a constant response of the kinetic energy fluctuations below a certain crossover frequency and then a $(1/f_m)$ decrease above. In terms of fluctuations, the velocity modulus fluctuations that we measure should depend linearly on kinetic energy fluctuations as long as the modulation amplitude is small (similarly to the injected power fluctuations which depend linearly on the velocity fluctuations as shown in figure 5*b*). Hence, our finding confirms the theoretical expectations for the response of the kinetic energy fluctuations.

The cascade time can be deduced from the response maxima of the injected power, as suggested by Heydt *et al.* (2003). The presence of extrema can be physically interpreted as the consequence of the delay between the energy injection and the energy dissipation. This delay is due to the time taken for of the energy from the large scale to be transferred to the small scales, say the cascade time, τ_C. Hence, for the modulated case, the phase shift between injection and dissipation is $2\pi f_m \tau_C$. Since the global kinetic energy variations of the flow obey

$$\frac{\mathrm{d}E(t)}{\mathrm{d}t} = \epsilon_I(t) - \epsilon_D(t), \tag{4.3}$$

extrema in the response are expected each time energy injection and energy dissipation are either in or out of phase. In our experiment, the amplitude response of the injected power presents two clear maxima at 2 and 5 Hz, separated by a minimum at 3.5 Hz. These appear to be exactly the response extrema found theoretically. The distance between the two maxima, $\Delta f_m = 3\,\mathrm{Hz}$, corresponds to the cascade time scale of the turbulence $\tau_C \approx 0.33\,\mathrm{s} = \frac{5}{3} 2\pi/\Omega_0$. Heydt *et al.* (2003) found that the characteristic time scale of turbulence τ_0 defined by the crossover in the turbulence intensity response (figure 3*a*) is also the time defined by consecutive maxima of resonances. This is exactly what we find: $\tau_C \approx \tau_0$. The characteristic time τ_C thus appears to be the first measurement of the cascade time of a stationary turbulent flow.

We now turn to the global (time-averaged) response of the system. Since the turbulence is stationary, the mean injected power equals the mean dissipated power.

The noteworthy feature in figure 6(*b*) is that the increase in the kinetic energy is simultaneous with a decrease in the injected (i.e. dissipated) power. This correlation is especially strong at the modulation frequency of 3.5 Hz, where both the mean dissipated power (figure 6*a*) and the injected power fluctuation (figure 3*b*) are minimum. However, for $f_m = 3.5$ Hz, it is surprising to find that the turbulence, even that perturbed by a large-scale modulation, dissipates the same amount of energy as the unmodulated turbulence (figure 6*a*) and contains more kinetic energy (figure 3*b*). From a practical point of view, this finding suggests that by modulating at a resonance frequency, possibly a more efficient mixing can be achieved with the same energy input.

In conclusion, we have succeeded in measuring the cascade time experimentally in a closed flow system. It would be worthwhile to compare our results to measurements in open flow geometries (i.e. grid or jet turbulence) which can be more homogeneous.

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